# ADM College for Women (Autonomous)

(Accredited with 'A' Grade by NAAC 4th Cycle)

(Affiliated to Bharathidasan University, Thiruchirappalli)

Nagapattinam – 611 001

Department : Economics

Course Name : Mathematical Methods in Economics

Class : I BA

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**UNIT I BASIC CONCEPTS**:

Mathematical Economics – Meaning – Importance – Uses – Straight line – Definition, Equation of Straight Line (given 2 points,1 point and Slope and 2 Intercept)

**UNIT II SET OPERATIONS**

Definition of Set, Types of sets, Operation of sets, Union of two or three sets, Intersection of two or three sets, Difference of two sets, Complement of a set – Venn Diagram (Page No.1 to 13 in Text Book) - Simple problems.

**UNIT III MATRIX**

Matrix – Definition, Types, Operations – Addition, Subtraction, Scalar Multiplication, (upto 3x3 order), Multiplication of two matrices (up to 3x3 order)- Define - order of a Matrix, Singular matrix and Non Singular Matrix – Simple problems.

**UNIT IV DETERMINANTS**

Determinants – Definition, Difference between Matrix and Determinants, Define Minors and Co- factors of each element of a determinant (up to 3x3 order). Simple problems (No properties of Determinants).

**UNIT V SOLVING SIMULTANEOUS EQUATIONS**

Definition of Cramer's rule – Uses of Cramer's Rule, Solving Simultaneous Equations using Cramer's Rule, (up to Three Variables). – Simple problem.

# UNIT –1

## BASIC CONCEPTS

### Mathematical Economics

Mathematical economics is a model of economics that utilizes math principles and methods to create economic theories and to investigate economic quandaries. Mathematics permits economists to conduct quantifiable tests and create models to predict future economic activity.

- Mathematical economics is a form of economics that relies on quantitative methods to describe economic phenomena.
- Although the discipline of economics is heavily influenced by the bias of the researcher, mathematics allows economists to explain observable phenomenon and provides the backbone for theoretical interpretation.

The marriage of statistical methods, mathematics and economic principles has created an entirely new branch of economics called Econometrics**.**

### Straight Line

A line is simply an object in geometry that is characterized under zero width object that extends on both sides.

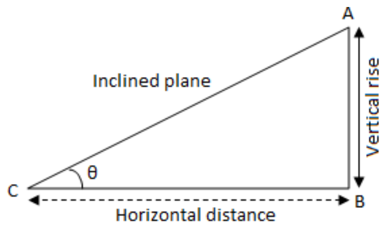
A straight line is just a line with no curves. So, a line that extends to both sides till infinity and has no curves is called a straight line.

Equation of line with slope ‘m’ and which passes through $(x_1, y_1)$ can be given as

$y - y_1 = m(x - x_1)$

### Concept of slope (or gradient):

If θ (≠ 90°) is the inclination of a straight line, then tan θ is called its slope or gradient. The slope of any inclined plane is the ratio between the vertical rise of the plane and its horizontal distance.




i.e., slope = vertical rise/horizontal distance

verticalrise/horizontaldistance = AB/BC = tan θ

Where θ is the angle which the plane makes with the horizontal

## Slope of a straight line:

The slope of a straight line is the tangent of its inclination and is denoted by letter 'm' i.e. if the inclination of a line is θ, its slope m = tan θ.

## Forms for the Equation of a Straight Line

Suppose that we have the graph of a straight line and that we wish to find its equation. (We will assume that the graph has x and y axes and a linear scale.)

The equation can be expressed in several possible forms.

**To find the equation of the straight line *in any form* we must be given either**:

- two points, $(x_1, y_1)$ and $(x_2, y_2)$, on the line; or
- one point, $(x_1, y_1)$, on the line and the slope, $m$; or
- The $y$ intercept, $b$, and the slope, $m$.

In the first case where we are given two points, we can find $m$ by using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Once we have one form we can easily get any of the other forms from it using simple algebraic manipulations. Here are the forms:

### 1. The slope-intercept form:

$y = m\,x + b$.

The constant $b$ is simply the $y$ intercept of the line, found by inspection. The constant $m$ is the slope, found by picking any two points $(x_1, y_1)$ and $(x_2, y_2)$ on the line and using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

### 2. The point-slope form:

$\mathbf{y - y_1 = m\,(x - x_1)}$.

$(x_1, y_1)$ is a point on the line. The slope $m$ can be found from a second point, $(x_2, y_2)$, and using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

### 3. The general form:

$a\,x + b\,y + c = 0$.

$a$, $b$ and $c$ are constants. This form is usually gotten by manipulating one of the previous two forms.

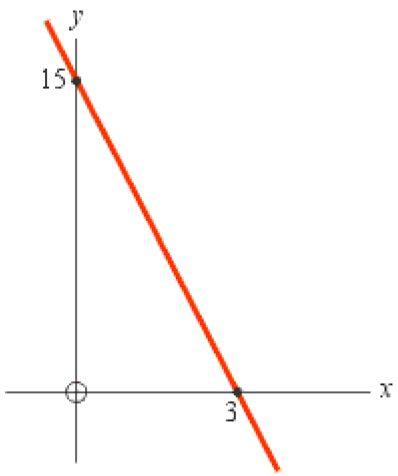
Note that any one of the constants can be made equal to 1 by dividing the equation through by that constant.

**4. The parametric form:**

$x = x_1 + t$
$y = y_1 + m\,t$

This form consists of a pair of equations; the first equation gives the $x$ coordinate and the second equation gives the $y$ coordinate of a point on the line as functions of a parameter $t$. $(x_1, y_1)$ is a known point on the line and $m$ is the slope of the line. Each value of $t$ gives a different point on the line.
For example when $t = 0$ then we get the point
$x = x_1$
$y = y_1$
or the ordered pair $(x_1, y_1)$, and when $t = 1$ then we get the point
$x = x_1 + 1$
$y = y_1 + m$
or the ordered pair $(x_1 + 1, y_1 + m)$, and so on.

**Example:** Show all of these forms for the straight line shown to the right.




**Solution:** Two points on this line are $(x_1, y_1) = (0, 15)$ and $(x_2, y_2) = (3, 0)$. Thus the $y$ intercept is $b = 15$ and the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = -5$$

**1.** To get the **slope-intercept form**, we simply substitute in the two values $m = -5$ and $b = 15$:
$y = m\,x + b.$

$y = -5\,x + 15.$

**2.** To get the **point-slope form**, we could use the point **(0, 15)** as “the” point together with $m = -5$:

$y - y_1 = m\,(x - x_1).$

$y - 15 = -5\,(x - 0)$ or simplifying:

$y - 15 = -5\,x.$

Or we could instead use the other point, **(3, 0)** and get:

$y - y_1 = m\,(x - x_1).$

$y - 0 = -5\,(x - 3)$ or simplifying:

$y = -5\,(x - 3).$

**3.** To get the **general form**, take any of these three forms found so far and distribute and collect all terms on the left-hand-side. The result is the same for all:

$5x + y - 15 = 0.$

Note that dividing both sides by, say 5, results in the equation

$x + 0.2y - 3 = 0,$

**4.** To get the **parametric form**, we could use the point (0, 15) as “the” point together with $m = -5$:

$x = x_1 + t$
$y = y_1 + mt$

$x = 0 + t$
$y = 15 - 5t$

With this choice, when $t = 0$ we are at the point (0, 15) and when $t = 3$ we are at the point (3, 0). We could instead use the other point, (3, 0) and get another parametric form:

$x = x_1 + t$
$y = y_1 + mt$

$x = 3 + t$
$y = 0 - 5t$

With this choice, when $t = 0$ we are at the point (3, 0) and when $t = -3$ we are at the point (0, 15). With either choice we will get all the points on the line as we let $t$ range through all values.

Example: Find the equation of the line that passes through the points (-2, 4) and (1, 2).

Solution**:**

We know general equation of a line passing through two points is:

y = mx + b, Here m = (2-4)/(1-(-2)) = -2/3

We can find the equation (by solving first for “b”) if we have a point and the slope. So we need to choose one of the points and use it to solve for b. Using the point (–2, 4), we get:

4 = (– 2/3)(–2) + b
4 = 4/3 + b
4 – 4/3 = b
b = 8/3
so, y = ( – 2/3 ) x + 8/3.

## Uses of Mathematics in Economics

Mathematical Economics is not a distinct branch of economics in the sense that public finance or international trade is. Rather, it is an approach to Economic analysis, in which the Economist makes use of mathematical symbols in the statement of the problem and also drawn up on known mathematical theorem to aid in reasoning. Mathematical economics insofar as geometrical methods are frequently utilized to derive theoretical results. Mathematical economics is reserved to describe cases employing mathematical techniques beyond simple geometry, such as matrix algebra, differential and integral calculus, differential equations, difference equations etc….

It is argued that mathematics allows economist to form meaningful, testable propositions about wide- range and complex subjects which could less easily be expressed informally. Further, the language of mathematics allows Economists to make specific, positive claims about controversial subjects that would be impossible without mathematics. Much of Economics theory is currently presented in terms of mathematical Economic models, a set of stylized and simplified mathematical relationship asserted to clarify assumptions and implications.

Mathematical economics is a model of economics that utilizes math principles and methods to create economic theories and to investigate economic quandaries. Mathematics permits economists to conduct quantifiable tests and create models to predict future economic activity.

Mathematical economics is a form of economics that relies on quantitative methods to describe economic phenomena. Although the discipline of economics is heavily influenced by the bias of the researcher, mathematics allows economists to explain observable phenomenon and provides the backbone for theoretical interpretation. The merge of statistical methods, mathematics and economic principles has created an entirely new branch of economics called Econometrics.

Mathematical economics is the application of mathematical methods to represent economic theories and analyze problems posed in economics. It allows formulation and derivation of key relationships in a theory with clarity, generality, rigor, and simplicity. By convention, the applied methods refer to those beyond simple geometry, such as differential and integral calculus, difference and differential equations, matrix algebra, and mathematical programming and other computational methods.

Mathematics allows economists to form meaningful, testable propositions about many wide-ranging and complex subjects which could not be adequately expressed informally. Further, the language of mathematics allows economists to make clear, specific, positive claims about controversial or contentious subjects that would be impossible without mathematics. Much of economic theory is currently presented in terms of mathematical economic models, a set of stylized and simplified mathematical relationships that clarify assumptions and implications.

Broad applications include:

1. Optimization problems as to goal equilibrium, whether of a household, business firm, or policy maker.
2. static (or equilibrium) analysis in which the economic unit (such as a household)or economic system (such as a market or the economy) is modeled as not changing
3. Comparative statics as to a change from one equilibrium to another induced by a change in one or more factors. dynamic analysis, tracing changes in an economic system over time, for example from economic growth
4. Formal economic modeling began in the 19th century with the use of differential calculus to represent and explain economic behavior, such as utility maximization, an early economic application of mathematical optimization. Economics became more mathematical as a discipline throughout the first half of the 20th century, but introduction of new and generalized techniques in the period around the Second World War, as in game theory, would greatly broaden the use of mathematical formulations in economics.
5. This rapid systematizing of economics alarmed critics of the discipline as well as some noted economists. John Maynard Keynes, Robert Heilbroner, Friedrich Hayek and others have criticized the broad use of mathematical models for human behavior, arguing that some human choices are irreducible to mathematics.

# UNIT-2

## SET OPERATIONS

### Definition of Set

A set refers to a collection of distinct objects. Any well-defined collection of mathematical objects can form a set.

These objects could be anything – from people's names to their ages/likes /dislikes; entities from simple number system to complex scientific data; from outcomes of a single dice roll or a coin toss to such experiments repeated 100s or 1000s of times.

- Collection of the names of the freedom fighters of India.
- Family of all natural numbers/whole numbers/odd numbers/even numbers/rational numbers/integers/real numbers.
- A group of possible outcomes of a dice roll or a coin toss.
- Collection of crucial data gathered by ISRO from MOM.
- A collection of day/night temperatures.

And there are many more such examples that will form a collection. The important point to notice here is that the rule that defines or law through which we collect or group objects should be universal.

### Conventions for Sets

The following are the conventions that are used here:

- Sets are usually denoted by a capital letter.
- The elements of the group are usually represented by small letters (unless specified separately.)
- If '**a**' is an element of '**A**', or if a "belongs to" A, it is written in the conventional notion by the use of the Greek symbol ϵ (Epsilon) between them a ϵ A.
- If **b** is not an element of Set A, b "does not belong to" A is written in the conventional notion by the use of the symbol ϵ (Epsilon with a line across it) between them – a ϵ A.
- Objects, elements, entities, members are all synonymous terms.

### Representations of a Set

Representation of Sets and its elements is done in the following two ways.

### Roster Form

In this form, all the elements are enclosed within braces **{}** and they are separated by commas (**,**). For example, a collection of all the numbers found on a dice N = {1, 2, 3, 4, 5, 6}.

**Properties of roster form**: –

- The order in which the elements are listed in the Roster form for any Set is immaterial. For example, V = {a, e, i, o, u} is same as V = {u, o, e, a, i}
- The dots at the end of the last element of any Set represent its infinite form and indefinite nature. For example, group of odd natural numbers = {1, 3, 5, …}
- In this form of representation, the elements are *generally* not repeated. For example, the group of letters forming the word POOL = {P, O, L}

More examples for Roster form of representation are:

- A = {3, 6, 9, 12}
- F = {2, 4, 8, 16, 32}
- H = {1, 4, 9, 16, …, 100}
- L = {5, 25, 125, 625}
- Y = {1, 1, 2, 3, 5, 8, …}

## Set Builder Form

In this form, all the elements possess a single common property which is NOT featured by any other element outside the Set. For example, a group of vowels in English alphabetical series.

The representation is done as follows. Let V be the collection of all English vowels, then

V = {x: x is a vowel in English alphabetical series.}

**Properties of Roster form:** –

- Colon (**:**) is a mandatory symbol for this type of representation.
- After the colon sign, we write the common characteristic property possessed by ALL the elements belonging to that Set and enclose it within braces.
- If the Set doesn't follow a pattern, its Set builder form cannot be written.

More examples for Set builder form of representation for a Set: –

- D = {x: x is an integer and – 3 < x < 19}
- O = {y: y is a natural number greater than 5}
- I = {f: f is a two – digit prime number less than 1000}
- R = {s: s is a natural number such that sum of its digits is 4}

- X = {m: m is a positive integer < 40}

Examples

Q1: Write the statements of representation of sets for an unbiased roll for a dice.

Solution:

- In Roster form – A = {1, 2, 3, 4, 5, 6}
- In Set Builder form – A = {x: x is natural number ≤ 6}

Q2: Write the statement of representation of set for a Fibonacci series in Roster form.

Solution: In Roster form, the Fibonacci series can be represented as: A = {1, 1, 2, 3, 5, 8, 13…}. Fibonacci series is a special category series that gets its next number by adding the previous two numbers.

Q3. What is a set in Mathematics?

When we look at sets in Mathematics, we see it is a well-defined collection comprising of various objects. For instance, the number 2, 4 and 6 are very different from each other when we look at them separately. However, when you consider them in a collective manner, they make up a single set of size three which we write as {2, 4, 6}

Q4. What is a proper set?

A proper subset of a set A is a subset of A which does not equal to A. Meaning to say if B is a proper subset of A then all elements of B are in A but A comprises a minimum of 1 element which is not present in B. For instance, if A = {5, 7, 9} then B = {5, 9}is a proper subset of A.

Q5. What is an example of a set?

As a set is a collection of different objects that contain common property, an example of a set will be dog, deer, lion and mouse is all animals. So, when you consider them collectively, they are a set.

Q6. What is the symbol of a set?

Sets are commonly denoted with a capital letter like A = {1, 2, 3, 4}. A set which contains no element is an empty or null set and we use { } or ∅ to denote this.

## Types of Sets

A Set is a well – defined collection of objects; depending on the objects and their characteristics, there are many types of Sets

## Empty or Null or Void Set

Any Set that does not contain any element is called the empty or null or void set. The symbol used to represent an empty set is – **{}** or **φ**. Examples:

- Let A = {x : 9 < x < 10, x is a natural number} will be a null set because there is NO natural number between numbers 9 and 10. Therefore, A = {} or φ
- Let W = {d: d > 8, d is the number of days in a week} will also be a void set because there are only 7 days in a week.

## Finite and Infinite Sets

Any set which is empty or contains a definite and countable number of elements is called a finite set. Sets defined otherwise, for uncountable or indefinite numbers of elements are referred to as infinite sets. Examples:

- A = {a, e, i, o, u} is a finite set because it represents the vowel letters in the English alphabetical series.
- B = {x : x is a number appearing on a dice roll} is also a finite set because it contains – {1, 2, 3, 4, 5, 6} elements.
- C = {p: p is a prime number} is an infinite set.
- D = {k: k is a real number} is also an infinite set.

## Equal and Unequal Sets

Two sets X and Y are said to be equal if they have exactly the same elements (irrespective of the order of appearance in the set). Equal sets are represented as X = Y. Otherwise, the sets are referred to as unequal sets, which are represented as X ≠ Y. Examples:

- If X = {a, e, i, o, u} and H = {o, u, i, a, e} then both of these sets are equal.
- If C = {1, 3, 5, 7} and D = {1, 3, 5, 9} then both of these sets are unequal.
- If A = {b, o, y} and B = {b, o, b, y, y} then also A = B because both contain same elements.

## Equivalent Sets

Equivalent sets are those which have an equal number of elements irrespective of what the elements are. Examples:

- A = {1, 2, 3, 4, 5} and B = {x : x is a vowel letter} are equivalent sets because both these sets have 5 elements each.
- S = {$1^2$, $2^2$, $3^2$, $4^2$, …} and T = {y : $y^2$ ϵ Natural number} are also equal sets.

## Singleton Set

These are those sets that have only a single element. Examples:

- E = {x : x ϵ N and $x^3$ = 27} is a singleton set with a single element {3}
- W = {v: v is a vowel letter and v is the first alphabet of English} is also a singleton set with just one element {a}.

## Universal Set

A universal set contains ALL the elements of a problem under consideration. It is *generally* represented by the letter U. Example:

- The set of Real Numbers is a universal set for ALL natural, whole, odd, even, rational and irrational numbers.

## Power Set

The collection of ALL the subsets of a given set is called a power set of that set under consideration. Example:

- A = {a, b} then Power set – P (A) = φ, {a}, {b} and {a, b}. If n (A) = m then generally, n [P (A)] = $2^m$

## Solved Examples for You

Question 1: If A = {x: x is an even natural number} and B = {y: y is the outcome of a dice roll}, determine the nature of the two sets.

Answer : A = {2, 4, 6, 8, 10, 12, 14, …} And B = {1, 2, 3, 4, 5, 6}. So, set A is an infinite set while set B is a finite set.

Question 2: If X = {1, 2, 3, 4, 5}, Y = {a, e, i, o, u} and Z = {u, o, a, i, e}; determine the nature of sets.

Answer : Since the pairs of sets X – Y, Y – Z as well as Z – X have the same number of elements, i.e. 5 they are EQUIVALENT sets. And sets Y and Z are also EQUAL sets because apart from having the number of elements the same, they also have the same elements, i.e. the alphabets of English vowel letters.

Question 3: What is the classification of sets in mathematics?

Answer: There are various kinds of sets like – finite and infinite sets, equal and equivalent sets, a null set. Further, there are a subset and proper subset, power set, universal set in addition to the disjoint sets with the help of examples.

Question 4: What are the properties of sets?

Answer: The fundamental properties are that a set can consist of elements and that two sets are equal, if and only if every element of each set is an element of the other; this property is referred to as the extensionality of sets.

Question 5: What are Finite and Infinite Sets?

Answer: Any set that is empty or consists of a definite and countable number of elements is referred to as a finite set. Sets explained otherwise, for uncountable or indefinite numbers of elements are called infinite sets.

Question 6: What is a universal set?

Answer: A universal set consists of all the elements of a problem under consideration. We generally represent it by the letter U. For instance, the set of Real Numbers is a universal set for all-natural, whole, odd, even, rational in addition to irrational numbers.

## Operations on Sets

Unlike the real world operations, mathematical operations do not require a separate no-contamination room, surgical gloves, and masks. But certainly, expertise to solve the problem, special tools, techniques, and tricks as well as knowledge of all the basic concepts are required to obtain a solution. Following are some of the operations that are performed on the sets:

- Union
- Intersection
- Difference
- Complement

## Union of Sets

Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12}. Then, A U B is represented as the set containing all the elements that belong to both the sets individually. Mathematically,

A U B = {x : x ϵ A or x ϵ B}

So, A U B = {2, 4, 6, 8, 10, 12},

Here the common elements are not repeated.

Properties of (A U B)

- Commutative law holds true as (A U B) = (B U A)
- Associative law also holds true as (A U B) U {C} = {A} U (B U C)

Let A = {1, 2} B = {3, 4} and C = {5, 6}
A U B = {1, 2, 3, 4} and (A U B) U C = {1, 2, 3, 4, 5, 6}
B U C = {3, 4, 5, 6} and A U (B U C) = {1, 2, 3, 4, 5, 6}
Thus, the law holds true and is verified.

- A U φ = A (Law of identity element)
- Idempotent Law – A U A = A
- Law of the Universal set (**U**): (A U **U**) = **U**

## Intersection of Sets

An intersection is the collection of all the elements that are **common** to all the sets under consideration. Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12} then A ∩ B or "A intersection B" is given by:

"A intersection B" or A ∩ B = {6, 8}

Mathematically, A ∩ B = {x : x ϵ A **and** x ϵ B}

Properties of the Intersection – A ∩ B

The intersection of the sets has the following properties:

- Commutative law – **A** ∩ **B = B**∩ **A**
- Associative law – (**A** ∩ **B)**∩ **C = A** ∩ **(B**∩ **C)**
- φ ∩ A = φ
- U ∩ A = A
- A∩ A = A; Idempotent law.
- Distributive law – **A** ∩ **(BU C) = (A** ∩ **B) U (A** ∩ **C)**

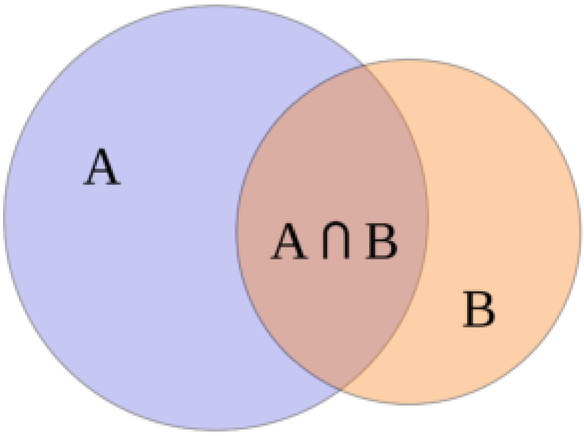
## Difference of Sets

The difference of set A and B is represented as:

A – B = {x : x ϵ A and x ϵ B}

Conversely, B – A = {x : x ϵ A and x ϵ B}

Let, A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8} then A – B = {1, 3, 5} and B – A = {8}. The sets (A – B), (B – A) and (A ∩ B) are **mutually disjoint sets**; it means that there is NO element common to any of the three sets and the intersection of any of the two or all the three sets will result in a null or void or empty set.




## Complement of Sets

If U represents the Universal set and any set A is the subset of A then the complement of set A (represented as A') will contain ALL the elements which belong to the Universal set U but NOT to set A.

Mathematically, **A' = U – A**

Alternatively, the complement of a set A, A' is the difference between the universal set U and the set A.

### Properties of Complement Sets

- A U A' = U
- A ∩ A' = φ
- De Morgan's Law – (A U B)' = A' ∩ B' OR (A ∩ B)' = A' U B'
- Law of double complementation : (A')' = A
- φ' = U
- U' = φ

Question 1: Let A = {1, 3, 5, 7} B = {5, 7, 9, 11} and C = {1, 3, 5, 7, 9, 11, 13} prove that:

**(A∩ B) U (A ∩ C) = A ∩ (BU C)**

Answer **:** B U C = {1, 3, 5, 7, 9, 11, 13}
A ∩ (B U C) = {1, 3, 5, 7}
Hence, A ∩ B = {5, 7}
A ∩ C = {1, 3, 5, 7}
(A∩ B) U (A∩ C) = {1, 3, 5, 7} … {Hence proved}

Question 2: Prove De Morgan's Law.

Answer : De Morgan's law is a very important and crucial concept in Set Theory. It serves numerous applications in the real world regarding Boolean algebra. The statement of the law reads as –

(A U B)' = A' ∩ B' OR (A ∩ B)' = A' U B'

We will prove this law by separately dealing with both the statements.

Case I: (A U B)' = A' ∩ B'

Let A = {Set of natural numbers ≤ 10} and B = {Even numbers ≤ 10}
So, A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = {2, 4, 6, 8, 10}
A U B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(A U B)' = φ … {Equation 1}
A' = φ and B' = {1, 3, 5, 7, 9}
A' ∩ B' = φ … {Equation 2}
By Equation 1 and 2 – **L.H.S. = R.H.S.**

Case II: (A ∩ B)' = A' U B'

Taking the same example, i.e. A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = {2, 4, 6, 8, 10}
A ∩ B = {2, 4, 6, 8, 10}
(A∩ B)' = {1, 3, 5, 7, 9} … {Equation 3}
A' = φ
B' = {1, 3, 5, 7, 9}
A' U B' = {1, 3, 5, 7, 9} … {Equation 4}
By Equation 3 and 4 – **L.H.S. = R.H.S.**

Question 3: What does ∩ mean?

Answer**:** In mathematics, the intersection of two given sets is the largest set that contains all the elements that are common to both the sets. In addition, the symbol for denoting intersection of sets is ∩, which is a common representation of sets.

Question 4: State the symbols of sets?

Answer: Basically there are four types of sets namely:

- **Subset-** A ⊆ B, where A is a subset of B and set A is part of set B.
- **Strict Subset/ Proper subset-** A⊂ B, where A is a subset of B, but A is not equal to B.
- **Not Subset-** A ⊄ B, here set A is not a subset of B.
- **Superset-** A ⊇ B, where A is a superset of B and set A includes set B.

Question 5: What is the intersection of two sets?

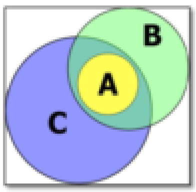
Answer**:** When two sets intersect they form a new set that contains all the elements of both the sets. Furthermore, we can write this intersection as A∩B.

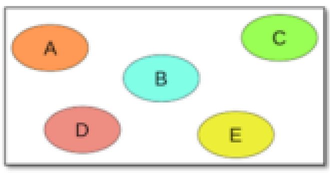
Question 6: What is the intersection of an empty set?

Answer: It is a set with no elements. Also, if there are no elements in at least one of the sets that we are trying to find then the two sets have no elements in common.

## Venn Diagrams

A Venn diagram is a diagrammatic representation of ALL the possible relationships between different sets of a finite number of elements. Venn diagrams were conceived around 1880 by John Venn, an English logician, and philosopher. They are extensively used to teach Set Theory. A Venn diagram is also known as **a** Primary diagram, Set diagram or Logic diagram.




**Representation of Sets in a Venn Diagram**

It is done as per the following:

- Each individual set is represented *mostly* by a circle and enclosed within a quadrilateral (the quadrilateral represents the finiteness of the Venn diagram as well as the Universal set.)
- Labeling is done for each set with the set's name to indicate difference and the respective constituting elements of each set are written within the circles.
- Sets having no element in common are represented separately while those having some of the elements common within them are shown with overlapping.

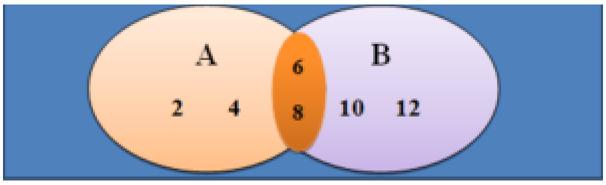
- The elements are written within the circle representing the set containing them and the common elements are written in the parts of circles that are overlapped.

## Operations on Venn Diagrams

Mathematical operations on sets like Union, Difference, Intersection, Complement, etc. we have operations on Venn diagrams that are given as follows:

## Union of Sets

Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12}. Represent A U B through a well-labeled Venn diagram.



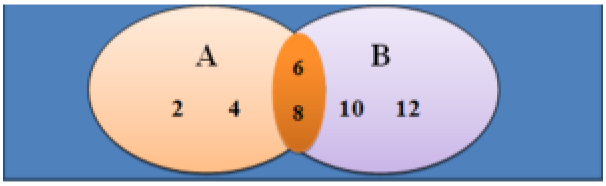

The orange colored patch represents the common elements {6, 8} and the quadrilateral represents A U B.

### Properties of A U B

- The commutative law holds true as A U B = B U A
- The associative law also holds true as (A U B) U C = A U (B U C)
- A U φ = A (Law of identity element)
- Idempotent Law – A U A = A
- Law of the Universal Set **U** – A U **U** = **U**

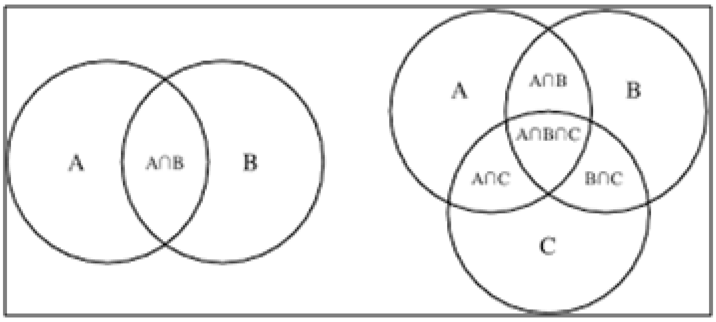
## The Intersection of Sets

An intersection is nothing but the collection of all the elements that are common to all the sets under consideration. Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12} then **A ∩ B** is represented through a Venn diagram as per following:




The orange colored patch represents the common elements {6, 8} as well as the **A ∩ B**. The intersection of 2 or more sets is the overlapped part(s) of the individual circles with the elements written in the overlapped parts.

Example:



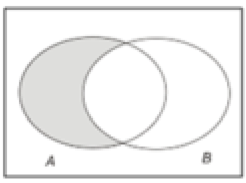

### Properties of A ∩ B

- Commutative law – **A ∩ B = B∩ A**
- Associative law – **(A ∩ B)∩ C = A ∩ (B∩ C)**
- φ ∩ A = φ
- U ∩ A = A
- A∩ A = A; Idempotent law.
- Distributive law – **A ∩ (B∩ C) = (A ∩ B) U(A ∩ C)**

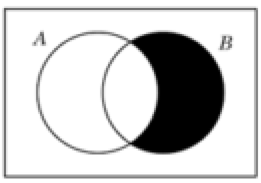
### Difference of Sets

The difference of set A and B is represented as: A – B = {x: x ϵ A and x ϵ B} {converse holds true for B – A}. Let, A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8} then A – B = {1, 3, 5} and B – A = {8}. The sets (A – B), (B – A) and (A ∩ B) are mutually disjoint sets.

It means that there is NO element common to any of the three sets and the intersection of any of the two or all the three sets will result in a null or void or empty set. **A – B** and **B – A** are represented through Venn diagrams as follows:




A – B



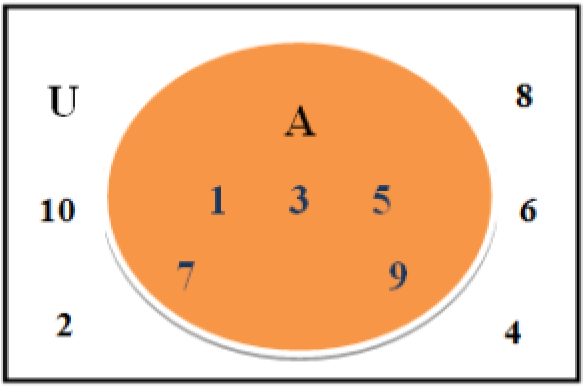

B – A

### Complement of Sets

If U represents the Universal set and any set A is the subset of A then the complement of set A (represented as A') will contain ALL the elements which belong to the Universal set U but NOT to set A.

Mathematically – **A' = U – A**

Alternatively, the complement of a set A, A' is the difference between the universal set U and the set A. Example: Let universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and set A = {1, 3, 5, 7, 9}, then complement of A is given as: A' = U – A = {2, 4, 6, 8, 10}




## Properties of Complement Sets

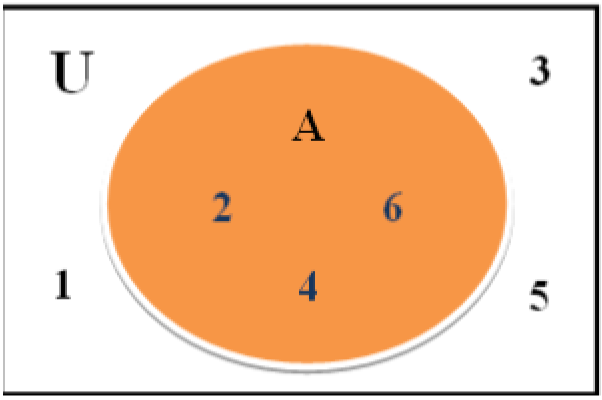
- A U A' = U
- A ∩ A' = φ
- De Morgan's Law – (A U B)' = A' ∩ B' OR (A ∩ B)' = A' U B'
- Law of double complementation : (A')' = A
- φ' = U
- U' = φ

## Venn Diagram

Solved Examples

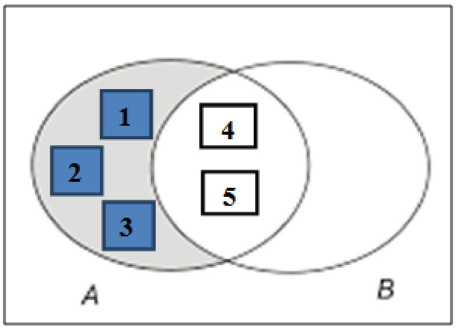
Question 1: Represent the Universal Set (U) = {x : x is an outcome of a dice's roll} and set A = {s : s ϵ Even numbers} through a Venn diagram.

Answer : Since, U = {1, 2, 3, 4, 5, 6} and A = {2, 4, 6}. Representing this with a Venn diagram we have:

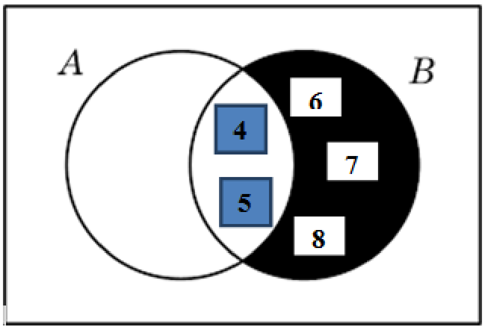


Here, A is a subset of U, represented as – **A ⊂ U** or
U is the superset of A, represented as – **U⊃ A**
If A = {1, 2, 3, 4, 5} and B = {4, 5, 6, 7, 8},
then represent A – B and B – A through Venn diagrams.
A – B = {1, 2, 3}
B – A = {6, 7, 8}
Representing them in Venn diagrams:

a. A – B




b. B – A




Question 2: Define the working of the Venn diagrams.
Answer: Venn diagrams permit the students to arrange the information visually so that they are able to see the relations between 2 or 3 sets of the items. They can then recognize the similarities and differences between them. A Venn diagram figure contains overlapping circles. Each circle comprises all the elements of one set.

Question 3: How can we describe a Venn diagram?
Answer: A Venn diagram is made with the overlapping circles. The inner of each circle shows a set of objects, or objects having something in common. The outside of the circle symbolizes all that a single set excludes.

Question 4: What is the medium of a Venn diagram known as?
Answer: The area of the joining of the 3 circles is the medium or middle point of a Venn diagram.

Question 5: What does the 'U' mean in a Venn diagram?
Answer: The '∪' in a Venn diagram represents the 'union of 2 sets'. Each sphere or ellipse in a Venn diagram represents a group.

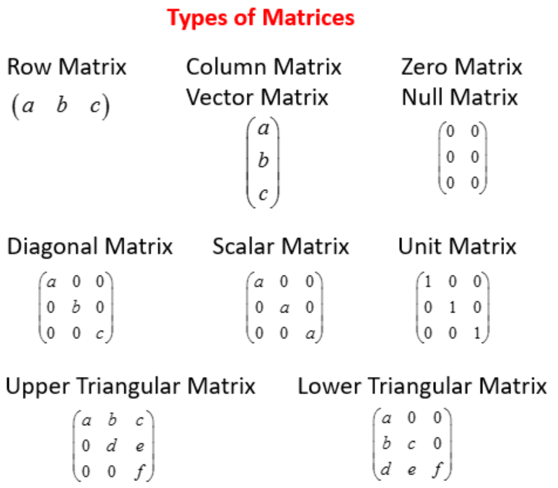
# UNIT-3

## Matrix

- Matrix - Definition,
- Matrix Types,
- Operations – Addition, Subtraction, Scalar Multiplication, (upto 3x3 order),
- Multiplication of two matrices (up to 3x3 order)
- Singular matrix and Non Singular Matrix – Simple problems.

## Matrix –Definition

A Matrix is a rectangular array of numbers. The size or dimension of a matrix is defined by the number of rows and columns it contains.




## Types of Matrices

A matrix may be classified by types. It is possible for a matrix to belong to more than one type.
A **row matrix** is a matrix with only one row.

Example: E is a row matrix of order 1 × 1

$$E = (4)$$

Example: B is a row matrix of order 1 × 3

$$B = (9 \quad -2 \quad 5)$$

A **column matrix** is a matrix with only one column.

Example: C is a column matrix of order 1 × 1

$$C = (3)$$

A column matrix of order 2 ×1 is also called a **vector** matrix.

Example: D is a column matrix of order 2 × 1

$$D = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

A **zero matrix** or a **null matrix** is a matrix that has all its elements zero.
Example: **O** is a zero matrix of order 2 × 3

$$O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A **square matrix** is a matrix with an equal number of rows and columns.
Example: T is a square matrix of order 2 × 2

$$T = \begin{pmatrix} 6 & 3 \\ 0 & 4 \end{pmatrix}$$

Example: V is a square matrix of order 3 × 3

$$V = \begin{pmatrix} 7 & 1 & 9 \\ 3 & 2 & 5 \\ 2 & 1 & 8 \end{pmatrix}$$

A **diagonal matrix** is a square matrix that has all its elements zero except for those in the diagonal from top left to bottom right; which is known as the **leading diagonal** of the matrix.
Example: B is a diagonal matrix.

$$B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

A **scalar matrix** is a diagonal matrix where all the diagonal elements are equal. For example:

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

An **upper triangular matrix** is a square matrix where all the elements located below the diagonal are zeros. For example:

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

A **lower triangular matrix** is a square matrix where all the elements located above the diagonal are zeros. For example:

$$\begin{bmatrix} 3 & 0 & 0 \\ -1 & 4 & 0 \\ 2 & 5 & 1 \end{bmatrix}$$

A **unit matrix** is a diagonal matrix whose elements in the diagonal are all ones.
Example: P is a unit matrix.

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## Matrix

A matrix is a rectangular grid of numbers or symbols that is represented in a row and column format. Each individual term of a matrix is known as elements or entries. The matrix is determined with the number of rows and columns. For example, a matrix with 2 rows and 3 columns is referred to as a 2 x 3 matrix.

Matrix can also have an even number of rows and columns; these are known as square matrix.

A row vector is a matrix made up on only one row of numbers, while a column vector is a matrix that is made up of only one column of numbers.

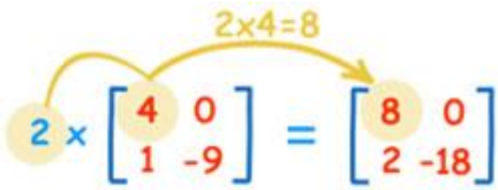
The matrices are usually enclosed in square or curved brackets. Each closed bracket is considered as a one matrix. These matrices are assigned a capital alphabet that represents the matrix.

The data in the matrix can be any type of number that we choose, including positive, negative, zero, fractions, decimals, symbols, alphabets, etc.

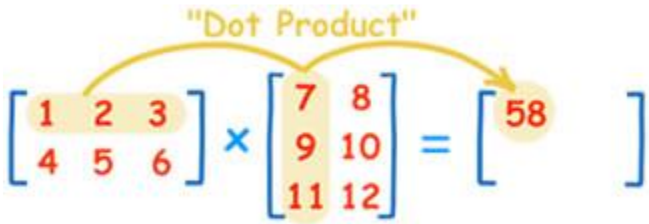
Matrices can be added, subtracted or multiplied. In case of addition, subtraction and multiplication of two matrices, the matrices must have the same number of rows and columns.

There are two forms of multiplication: scalar multiplication and multiplication of a matrix by another matrix.

**Scalar matrix** includes multiplying a matrix with a single number.




Multiplication of two matrices with each other requires solving them in a 'dot product', where a single row is multiplied with a single column. The resulting figures are then added up. The result of th first multiplication would be 1 x 7 + 2 x 9 + 3 x 11 = 58.




There are various different kinds of matrices: Square, diagonal and identity. A square matrix is a matrix that has the same number of rows and columns i.e.: 2x2, 3x3, 4x4, etc.

A diagonal matrix is a square matrix that has zeros as elements in all places, except in the diagonal line, which runs from top left to bottom right.

An identity matrix is a diagonal matrix that has all diagonal elements equal to 1.

Matrices are applied prominently in linear transformation, required for solving linear functions. Other fields that include matrices are classical mechanics, optics, electromagnetism, quantum mechanics, and quantum electrodynamics. It is also used in computer programming, graphics and other computing algorithms.

## Matrix Operations

### Matrix Addition

A B

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad = \begin{pmatrix} 7 & 1 & 9 \\ 3 & 2 & 5 \\ 2 & 1 & 8 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 10 & 1 & 9 \\ 3 & 10 & 5 \\ 2 & 1 & 10 \end{pmatrix}$$

### Matrix Subtraction

A B

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad = \begin{pmatrix} 7 & 1 & 9 \\ 3 & 2 & 5 \\ 2 & 1 & 8 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -4 & -1 & -9 \\ -3 & 6 & -5 \\ -2 & -1 & -8 \end{pmatrix}$$

Example:

### Matrix Multiplication:

1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 1\times2+2\times1 & 1\times0+2\times2 \\ 3\times2+4\times1 & 3\times0+4\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

**2.** $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 2\times1+0\times3 & 2\times2+0\times4 \\ 1\times1+2\times3 & 1\times2+2\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 7 & 10 \end{bmatrix}$$

## Singular matrix and Non Singular Matrix

A matrix can be singular, only if it has a determinant of zero. A matrix with a non-zero determinant certainly means a non-singular matrix.

### Singular Matrix - Definition

Singular matrix:

A square matrix whose determinant is 0 is called singular matrix.

Examples: $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

### Non Singular Matrix - Definition

Non singular matrix:

A square matrix that is not singular, i.e. one that has matrix inverse. Non singular matrices are sometimes also called regular matrices. A square matrix is non singular if its determinant is non zero.

Example: $\begin{pmatrix} 532 \\ 197 \\ 556 \end{pmatrix}$

# Unit -4

## Determinants

- Determinants – Definition
- Difference between Matrix and Determinants
- Define Minors and Co- factors of each element of a determinant (up to 3x3 orders).
- Simple problems (No properties of Determinants).
-

### Definition

A determinant is a component of a square matrix and it cannot be found in any other type of matrix. A determinant is a real number that can be informally considered as the result of solving a square matrix. Determinant is denoted as det (matrix A) or |A|. It may seem like the absolute value of A, but in this case it refers to determinant of matrix A. The determinant of a square matrix is the product of the elements on the main diagonal minus the product of the elements off the main diagonal.

Let's assume the example of matrix B:

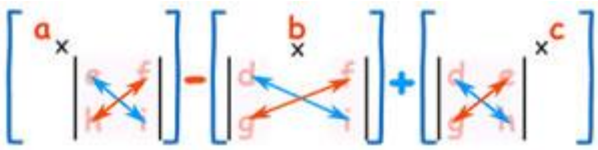
$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$$

The determinant of matrix B or |B| would be 4 x 6 – 6 x3.

This would give the determinant as 6.

For a 3x3 matrix, a similar pattern would be used.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$



## Properties of determinants:

- The determinant is a real number, it is not a matrix.
- The determinant can be a negative number.
- It is not associated with absolute value at all except that they both use vertical lines.
- The determinant only exists for square matrices (2×2, 3×3, ... n×n). The determinant of a 1×1 matrix is that single value in the determinant.
- The inverse of a matrix will exist only if the determinant is not zero.

## Difference between Matrix and a Determinant

1. Matrices do not have definite value, but determinants have definite value.

2. In a Matrix the number of rows and columns may be unequal, but in a Determinant the number of rows and columns must be equal.

3. The entries of a Matrix are listed within a large paranthesis (large braces), but in a determinant the entries are listed between two strips (i.e. between two vertical lines).

4. Let A be a Matrix. Matrix kA is obtained by multiplying all the entries of the Matrix by k. Let be any Determinant. is obtained by multiplying 'every entry of a row ' or 'every entry of a column' by k

A matrix or matrices is a rectangular grid of numbers or symbols that is represented in a row and column format. A determinant is a component of a square matrix and it cannot be found in any other type of matrix.

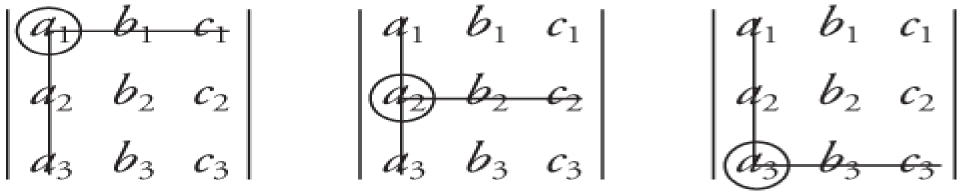
- Matrices and determinants are important concepts in linear mathematics.
- These concepts play a huge part in linear equations are also applicable to solving real-life problems in physics, mechanics, optics, etc.
- A matrix is a grid of numbers, symbols or expressions that is arranged in a row and column format.
- A determinant is a number that is associated with a square matrix.
- These two terms can become quite confusing for people that are just learning these concepts.

The determinant of a 3 × 3 matrix can be defined as shown in the following.

minor determinants

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \underset{\text{subtract}}{-} a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} \underset{\text{add}}{+} a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

Each minor determinant is obtained by crossing out the first column and one row.

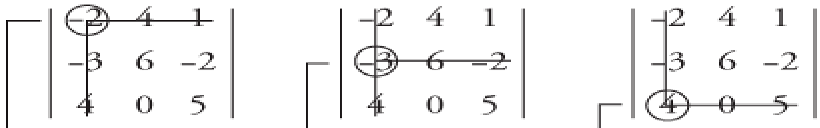


Example 1

Evaluate the following determinant.

$$\begin{vmatrix} -2 & 4 & 1 \\ -3 & 6 & -2 \\ 4 & 0 & 5 \end{vmatrix}$$

First find the minor determinants.



$$-2\begin{vmatrix} 6 & -2 \\ 0 & 5 \end{vmatrix} \underset{\text{subtract}}{-} -3\begin{vmatrix} 4 & 1 \\ 0 & 5 \end{vmatrix} \underset{\text{add}}{+} 4\begin{vmatrix} 4 & 1 \\ 6 & -2 \end{vmatrix}$$

$$-2(30-0) \quad - \quad -3(20-0) \quad + \quad 4(-8-6)$$

$$-60 \quad - \quad -60 \quad + \quad -56$$

$$-60 \quad +60 \quad -56 \quad = \quad -56$$

The solution is $\begin{vmatrix} -2 & 4 & 1 \\ -3 & 6 & -2 \\ 4 & 0 & 5 \end{vmatrix} = -56$

## Cofactor of a Determinant

The cofactor is defined as the signed minor. Cofactor of an element $a_{ij}$, denoted by $A_{ij}$ is defined by $A = (-1)^{i+j} M$, where M is minor of $a_{ij}$.

Note

- We note that if the sum i+j is even, then $A_{ij} = M_{ij}$, and that if the sum is odd, then $A_{ij} = -M_{ij}$.
- Hence, the only difference between the related minor entries and cofactors may be a sign change or nothing at all.
- Whether or $A_{ij} = M_{ij}$ or $A_{ij} = -M_{ij}$
- has a pattern for square matrices as illustrated:

For example $C_{12} = -M_{12}$. Of course, if you forget, you can always use the formula $C_{ij} = (-1)^{i+j} M_{ij}$,

Here, $C_{12}=(-1)^{1+2} M_{ij} = (-1)^3 M_{ij} = -M_{ij}$

**Minors and Cofactors** of all the elements of the determinant

$$\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$$

Solution: Minor of the element $a_{ij}$ is $M_{ij}$.
Here $a_{11} = 1$. So $M_{11}$ = Minor of $a_{11} = 3$
$M_{12}$ = Minor of the element $a_{12} = 4$
$M_{21}$ = Minor of the element $a_{21} = -2$
$M_{22}$ = Minor of the element $a_{22} = 1$

**Now, cofactor of $a_{ij}$ is $A_{ij}$. So,**
**$A_{11} = (-1)^{1+1}$, $M_{11} = (-1)^2 (3) = 3$**
**$A_{12} = (-1)^{1+2}$, $M_{12} = (-1)^3 (4) = -4$**
**$A_{21} = (-1)^{2+1}$, $M_{21} = (-1)^3 (-2) = 2$**
**$A_{22} = (-1)^{2+2}$, $M_{22} = (-1)^4 (1) = 1$**

# UNIT -5

## SOLVING SIMULTANEOUS EQUATIONS

- Definition of Cramer's rule
- Uses of Cramer's Rule
- Solving Simultaneous Equations using Cramer's Rule, (up to Three Variables). Simple problem.

## Cramer's Rule

To use determinants to solve a system of three equations with three variables (Cramer's Rule), say *x*, *y*, and *z*, four determinants must be formed following this procedure:

1. Write all equations in standard form.
2. Create the denominator determinant, *D*, by using the coefficients of *x*, *y*, and *z* from the equations and evaluate it.
3. Create the *x*-numerator determinant, $D_x$, the *y*-numerator determinant, $D_y$, and the *z*-numerator determinant, $D_z$, by replacing the respective *x*, *y*, and *z* coefficients with the constants from the equations in standard form and evaluate each determinant.

The answers for *x*, *y*, and *z* are as follows: $x = \frac{D_x}{D},\ y = \frac{D_y}{D},\ z = \frac{D_z}{D}$

Example

Solve this system of equations, using Cramer's Rule.

$$\begin{cases} 3x + 2y - z = 2 \\ 2x - y - 3z = 13 \\ x + 3y - 2z = 1 \end{cases}$$

Find the minor determinants.

*x*-coefficients, *y*-coefficients, *z*-coefficients

$$D = \begin{vmatrix} 3 & 2 & -1 \\ 2 & -1 & -3 \\ 1 & 3 & -2 \end{vmatrix} = 3\begin{vmatrix} -1 & -3 \\ 3 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ -1 & -3 \end{vmatrix}$$

$$= 3[2-(-9)] - 2[-4-(-3)] + 1(-6-1)$$

$$= 3(11) - 2(-1) + 1(-7)$$

$$= 33 + 2 - 7 = 28$$

Use the constants to replace the *x*-coefficients.

constants replacing the *x*-coefficients

$$D_x = \begin{vmatrix} 2 & 2 & -1 \\ 13 & -1 & -3 \\ 1 & 3 & -2 \end{vmatrix} = 2\begin{vmatrix} -1 & -3 \\ 3 & -2 \end{vmatrix} - 13\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ -1 & -3 \end{vmatrix}$$

$$= 2[2 -(-9) -13[-4 -(-3)] +1(-6 -1)$$

$$= 2(11) - 13(-1) + 1(-7)$$

$$= 22 + 13 - 7 = 28$$

Use the constants to replace the *y*-coefficients.

constants replacing the *y*-coefficients

$$D_y = \begin{vmatrix} 3 & 2 & -1 \\ 2 & 13 & -3 \\ 1 & 1 & -2 \end{vmatrix} = 3\begin{vmatrix} 13 & -3 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 13 & -3 \end{vmatrix}$$

$$= 3[-26-(-3)] -2[-4 -(-1)] +1[-6-(-13)]$$

$$= 3(-23) - 2(-3) + 1(7)$$

$$= -69 + 6 + 7 = -56$$

Use the constants to replace the *z*-coefficients.

constants replacing the *z*-coefficients

$$D_z = \begin{vmatrix} 3 & 2 & 2 \\ 2 & -1 & 13 \\ 1 & 3 & 1 \end{vmatrix} = 3\begin{vmatrix} -1 & 13 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ -1 & 13 \end{vmatrix}$$

$$= 3(-1-39) - 2(2-6) + 1[26 -(-2)]$$

$$= 3(-40) - 2(-4) + 1(28)$$

$$= -120 + 8 + 28 = -84$$

Therefore,

$$x = \frac{D_x}{D} = \frac{28}{28} = 1,\ \ y = \frac{D_y}{D} = -\frac{56}{28} = -2,\ z = \frac{D_z}{D} = -\frac{84}{28} = -3$$

The check is left to you. The solution is $x = 1$, $y = -2$, $z = -3$.

If the denominator determinant, *D*, has a value of zero, then the system is either inconsistent or dependent. The system is dependent if all the determinants have a value of zero. The system is inconsistent if at least one of the determinants, $D_x$, $D_y$, or $D_z$, has a value not equal to zero and the denominator determinant has a value of zero.

## Cramer's Rule: The method of determinants

A system of two equations in two unknowns has this form:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

The *a*'s are the coefficients of the *x*'s. The *b*'s are the coefficients of the *y*'s. The following is the matrix of those coefficients.

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

The *number* $a_1b_2 - b_1a_2$ is called the determinant of that matrix.

$$\det \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - b_1a_2$$

Let us denote that determinant by D.

Now consider this matrix in which the *c*'s replace the coefficients of the *x*'s:

$$\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

Then the determinant of that matrix -- which we will call D*x* is $c_1b_2 - b_1c_2$

And consider this matrix in which the *c*'s replace the coeffients of the *y*'s:

$$\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

The determinant of that matrix -- D*y* is $a_1c_2 - c_1a_2$

Cramer's Rule then states the following:

In every system of two equations in two unknowns in which the determinant
D is not 0,

$$x = \frac{Dx}{D}$$

$$y = \frac{Dy}{D}$$

Use Cramer's Rule to solve this system of equations:

$5x + 3y = -11$
$2x + 4y = -10$

Solution.

$$D = \det\begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} = 5\cdot 4 - 3\cdot 2$$

$$= 20 - 6 = 14$$

$$Dx = \det\begin{vmatrix} -11 & 3 \\ -10 & 4 \end{vmatrix} = -11\cdot 4 - 3\cdot -10$$

$$= -44 + 30 = -14$$

$$Dy = \det\begin{vmatrix} 5 & -11 \\ 2 & -10 \end{vmatrix} = 5\cdot -10 - (-11)\cdot 2$$

$$= -50 + 22$$

$$= -28.$$

Therefore,

$$x = \frac{Dx}{D} = \frac{-14}{14} = -1.$$

$$y = \frac{Dy}{D} = \frac{-28}{14} = -2.$$

Use Cramer's Rule to solve these simultaneous equations.

$3x - 5y = -31$

$2x + \ y = 1$

$$D = \det\begin{vmatrix} 3 & -5 \\ 2 & 1 \end{vmatrix} = 3\cdot 1 - (-5)\cdot 2$$

$$= 3 + 10 = 13$$

$$Dx = \det\begin{vmatrix} -31 & -5 \\ 1 & 1 \end{vmatrix} = -31\cdot 1 - (-5)\cdot 1$$

$$= -31 + 5 = -26$$

$$Dy = \det\begin{vmatrix} 3 & -31 \\ 2 & 1 \end{vmatrix} = 3\cdot 1 - (-31)\cdot 2$$

$$= 3 + 62 = 65$$

$$x = \frac{Dx}{D} = \frac{-26}{13} = -2.$$

$$y = \frac{Dy}{D} = \frac{65}{13} = 5.$$

## Advantages and Disadvantages of Cramer's Rule

### Advantages

1) Cramer's Rule is that you can find the value of x, y, or z without having to know any of the other values of x, y, or z.

For example, if you needed to find just the value of y, Cramer's Rule would work well.

2) Cramer's Rule is that if any of the values of x, y, or z is fractions, you do not have to plug in a fraction to find the other values. Each value can be found independently.

### Disadvantages

One of the only disadvantages to using Cramer's rule is if the value of D is zero then Cramer's Rule will not work because you cannot divide by zero. However, if the value of D is zero then you know that the solution is either "No Solution" or "Infinite Solutions". You will have to use a different technique such as Addition/Elimination to find out whether the answer is "No Solution" or "Infinite Solutions.